

ISI ENTRANCE → MOCK TEST

Part 2 → Subjective Type Question

- **Answer all the questions.**
- **This question booklet contains 8 Questions.**
- **Do not write anything on the question paper.**
- **Open this question paper only when you are told to do so.**
- **Marking Scheme – Each question carries 10 marks**
- **All the rough works has to be done on the answer sheet.**
- **You are not suppose to use any calculators or mobile phones, if found, your paper will be immediately cancelled.**

TOTAL MARKS: 80 | Time Limit: 2 hr

Q1. Prove that when n is a sufficiently large positive integer there exists a finite set S of prime numbers such that the sum of $\left[\frac{n}{p}\right]$ over $p \in S$ is equal to n .

Q2. Find all pairs (x, y) of integers such that

$$x^2 + 3xy + 4006(x + y) + 2003^2 = 0$$

Q3. Suppose that $f: [0,1] \rightarrow \mathbb{R}$ has a continuous second order derivative with $f''(x) > 0$ on $(0, 1)$, suppose that $f(0) = 0$. Choose $a \in (0, 1)$ such that $f'(a) < f(1)$. Show that there is a unique $b \in (a, 1)$ such that $f'(a) = \frac{f(b)}{b}$

Q4. Show that— $\int_0^\pi \left| \frac{\sin nx}{x} \right| dx \geq \frac{2}{\pi} \left(1 + \frac{1}{2} + \dots + \frac{1}{n} \right)$

Q5. Suppose $P(x)$ is a polynomial of degree n such that

$$P(k) = \frac{k}{k+1} \text{ for } k = 0, 1, \dots, n$$

Find the value of $P(n+1)$

Q6. Evaluate the given limit –

$$\lim_{n \rightarrow \infty} \frac{1}{2n} \log \binom{2n}{n}$$

Q7. Prove that

$$a(b-c)^2 + b(c-a)^2 + c(a-b)^2 + 4abc \geq a^3 + b^3 + c^3$$

where a, b, c denotes the three sides of a triangle.

Q8. Three identical black balls, four identical red balls and five identical white balls are to be arranged in a row. Find the number of ways that can be done if all the balls with the same colour do not form a single block.
