

ELIXIIT ACADEMY
– The Toppers' Point



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ISI CHALLENGING PROBLEMS WORKSHEET

1. Find all the natural numbers N such that the reduced residue system consisting of least positive residues modulo N form an arithmetic progression.
2. Find all functions $f: \mathbb{R} \rightarrow \mathbb{R}$ such that

$$x^2y^2(f(x+y) - f(x) - f(y)) = 3(x+y)f(x)f(y)$$

for all real numbers x and y .

3. P is a point in the minor arc AB of the circumcircle of the regular pentagon $ABCDE$; prove that $PA + PB + PD = PC + PE$

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4. Three identical black balls, four identical red balls and five identical white balls are to be arranged in a row. Find the number of ways that can be done if all the balls with the same colour do not form a single block.

5. A function $f: [0, 1] \rightarrow [0, 1]$ is continuous on $[0, 1]$. Prove that there exists a point c in $[0, 1]$ such that $f(c) = c$.

6. A function $f: [0, \infty) \rightarrow \mathbb{R}$ is continuous on $[0, \infty)$ and $\lim_{n \rightarrow \infty} f(x) = 0$. Prove that f is uniformly continuous on $[0, \infty)$.

7. Let $f: [a, b] \rightarrow \mathbb{R}$ be differentiable at a point c of $[a, b]$. Let $(\alpha_n), (\beta_n)$ be two sequences satisfying $a < \alpha_n < c < \beta_n < b$ for all $n \in \mathbb{N}$, both converge to c . Prove that

$$\lim_{n \rightarrow \infty} \frac{f(\beta_n) - f(\alpha_n)}{\beta_n - \alpha_n} = f'(c)$$

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