ELIXIIT ACADEMY
— The Toppers' Point

IIT-JEE MONTHLY UNIT TEST

TOPIC→ DIFFERENTIAL CALCULUS

OBJECTIVE TYPE QUESTIONS

- **Answer all the questions.**
- **This question paper contains 15 Questions. 10 Objective Type Questions and 5 Numerical Answer Type (NAT) Questions**
- **Do not write anything on the question paper.**
- **Mark/write the correct answer on the OTA sheet provided to you.**
- **Open this question paper only when you are told to do so.**
- **Marking Scheme for Objective Type Questions — +4 will be awarded for every correct answer and −1 will be deducted for every incorrect answer and 0 will be awarded for leaving a question unanswered.**
- **Marking Scheme for Numerical Answer Type (NAT) Questions: +4 will be awarded for every correct answer and 0 will be awarded for every incorrect answer/leaving unanswered.**
- **One or more than one option may be correct**
- **Do not make any mistake while filling the OTA sheet because no new/extra OTA sheet will be provided to the students.**
- **If any mistake is found on the OTA sheet, then the paper will be immediately cancelled.**
- **You are not suppose to use any calculators or mobile phones, if found, your paper will be immediately cancelled.**

FULL MARKS → 60 | TIME LIMIT→ 1 Hour

PART 1→OBJECTIVE TYPE QUESTIONS;

NOTE THAT ONE OR MORE THAN ONE OPTIONS MAY BE CORRECT IN THIS SECTION

Q1. Let $f: R \to (0, \infty)$ *and* $g: R \to R$ be twice differentiable functions such that f''& g'' are continuous functions on R. Suppose $f'(2) = g(2) = 0, f''(2) \neq 0$. If $\lim_{x\to 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$, then

(a) f has a local maximum at x = 2

(b) f has a local minimum at x = 2

(c) $f''(2) > f(2)$

(d) $f(x) - f''(x) = 0$ for at least one $x \in R$.

Q2. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then, $y'(1)$ equals.

(a) -1 (b) 1 (c) log 2 (d) –log 2

Q3. If a, b, c be non zero real numbers such that $\int_0^1 (1 + \cos^8 x)\,(ax^2 + bx + c)dx = \int_0^2 (1 + \cos^8 x)(ax^2 + bx + c)dx = 0$, then the equation $ax^2 + bx + c = 0$ will have

(a) one root between 0 and 1 and other root between 1 and 2

(b) both roots between 0 and 1.

(c) both the roots between 1 and 2

(d) none of these

Q4. If the tangent at a point P, with parameter t, on the curve $x = 4t^2 + 3,\ y = 8t^3 - 1,\ t \in R$, meets the curve again at Q, then the coordinates of Q are

(a) $(t^2+3, -t^3-1)$

(b) (t^2+3, t^3-1)

(c) $(16t^2+3, -64t^3-1)$

(d) $(4t^2+3, -8t^3-1)$

Q5. The equation of a curve is $y = f(x)$. The tangents at (1, f(1)), (2, f(2)) and (3, f(3)) make angles $\frac{\pi}{6}, \frac{\pi}{3}$ & $\frac{\pi}{4}$ respectively with the positive direction of the X-axis. Then the value of

$$\int_2^3 f'(x)\, f''(x)\, dx + \int_1^3 f''(x)\, dx$$

is equal to

(a) $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (c) 0 (d) None of these

Q6. Out of a circular sheet of paper of radius a, a sector with central angle θ is cut out and folded into the shape of a conical funnel. The volume of this funnel is maximum when θ equals.

(a) $\frac{2\pi}{\sqrt{2}}$ (b) $2\pi\sqrt{\frac{2}{3}}$ (c) $\frac{\pi}{2}$ (d) π

Q7. The curve $y = \frac{2x}{1+x^2}$ has

(a) exactly three points of inflection separated by a point of maximum and a point of minimum.

(b) exactly two points of inflection with a point of maximum lying between them.

(c) exactly two points of inflection with a point of minimum lying between them.

(d) exactly three points of inflection separated by two points of maximum.

Q8. The set of all x for which the function $f(x) = \log_{1/2}(x^2 - 2x - 3)$ is defined and monotone increasing is

(a) $(-\infty, 1)$ (b) $(-\infty, -1)$ (c) $(1, \infty)$ (d) $(3, \infty)$

Q9. Let $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$, where a_0, a_1, a_2 *and* a_3 are constants. Then which one of the statement is correct

(a) f(x) is differentiable at x=0 whatever be a_0, a_1, a_2 *and* a_3
(b) f(x) is differentiable at x=0 whatever be a_0, a_1, a_2 *and* a_3
(c) If f(x) is differentiable at x=0, then $a_1 = 0$
(d) If f(x) is differentiable at x=0, then $a_1 = 0,\ \ a_3 = 0$

Q10. The value of k for which

$$f(x) = \begin{cases} \frac{x^{2^{32}} - 2^{32}x + 4^{16} - 1}{(x-1)^2}, & x \neq 0 \\ k & x = 1 \end{cases}$$

is continuous at x =1 is

(a) $2^{63} - 2^{31}$ (b) $2^{65} - 2^{33}$ (c) $2^{62} - 2^{31}$ (d) $2^{65} - 2^{31}$

PART 2→NUMERICAL ANSWER TYPE (NAT) QUESTIONS:

Q11. Let P be a point in the first quadrant lying on the ellipse $\frac{x^2}{8} + \frac{y^2}{8} = 1$. Let AB be the tangent at P to the ellipse meeting the x-axis at A and y-axis at B. If O is the origin, then minimum possible area of the triangle OAB is ____________

Q12. $\lim_{h\to 0} \frac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)}$ given that $f'(2) = 6$ *and* $f'(1) = 4$ equals ________.

Q13. If the parabola $y = ax^2 + bx + c$ has vertex at (4, 2) and $a \in [1, 3]$, then the difference between the extreme values of abc is equal to ___________.

Q14. If $f(x) = \lim_{n\to\infty} \sum_{r=0}^{n} \frac{\tan(\frac{x}{2^{r+1}}) + \tan^3 \frac{x}{2^{r+1}}}{1 - tan^2(\frac{x}{2^{r+1}})}$ then $\lim_{x\to 0} \frac{f(x)}{x}$ is equal to ______

Q15. Let $f(x)$ be a polynomial satisfying $\left(f(\alpha)\right)^2 + \left\{\left(f(a)\right)^2\right\}$. Then,

$\lim_{x\to\alpha} \frac{f(x)}{f'(x)} \left[\frac{f'(x)}{f(x)}\right]$ is equal to __________(Here [.] denotes the greatest integer function)